

# Singularity in self-energy and composite fermion excitations of interacting electrons

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We propose that a composite fermion operator  $f_{i\sigma}(2n_{i\bar{\sigma}} - 1)$  could have coherent excitations, where  $f_{i\sigma}$  is the fermion operator for interacting electrons and  $n_{i\bar{\sigma}}$  is the number operator of the opposite spin. In the two-impurity Anderson model, it is found that the excitation of this composite fermion has a pseudogap in the Kondo regime, and has a finite spectral weight in the regime where the excitation of the regular fermion  $f_{i\sigma}$  has a pseudogap. In the latter regime, the self-energy of  $f_{i\sigma}$  is found to be singular near Fermi energy. We argue that this composite fermion could develop a Fermi surface with Fermi liquid behaviors but “hidden” from charge excitations in lattice generalizations. We further illustrate that this type of excitations is essential in addressing the pseudogap state and unconventional superconductivity.

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**Introduction.** The central issue in strongly correlated electron systems is on how the strong Coulomb interaction leads to many fascinating phenomena such as magnetic quantum phase transitions (QPTs), Mott transitions, and unconventional superconductivity in strongly correlated electron systems. In cuprate superconductors, a hotly debated issue is on the pseudogap state [1], as a doped Mott insulator, where the single-particle excitations are suppressed near the Fermi energy. Meanwhile, It is suggested the being in the vicinity of a Mott transition plays an important role in understanding the superconductivity in iron-based superconductors [2]. The “Kondo-breakdown” QPT in heavy fermions can also be viewed as a Mott transition in  $f$ -electrons [3]. The pseudogap physics, though not explicitly observed in the latter two systems, is expected to be present as well [4]. To understand the origin of a pseudogap therefore presents a key role in understanding other emerging properties of correlated electrons, in particular, superconductivity. The major challenge, which prevents such a understanding, is the lack of proper theoretical methods to treat strong Coulomb interactions between  $d$  or  $f$  electrons in these materials, as captured by some fundamental theoretical models such as the Hubbard model and the periodic Anderson lattice model. In this sense, the dynamical mean-field theory (DMFT) and its cluster extensions have made a great stride in this direction, by mapping the lattice problem into a self-consistent single (or a cluster) Anderson impurity model [5]. In cluster-DMFT solutions to the Hubbard model, a pseudogap is typically identified [6]. Further it has been shown that the pseudogap is associated with a singularity in self-energies [7]. However, there are no explicit explanations or detailed analyses on this behavior.

In this Letter, we provide such an analysis from a minimal two-impurity Anderson model [8]. This model captures both the local quantum fluctuations such as the Kondo dynamics, and the short-range spatial fluctuations, such as the inter-site spin or charge correlations, which are also the key fluctuations in lattice systems. This model is subject to *exact* numerical solutions, such as the numerical renormalization group (NRG) method [9] as we adopt below. With the advantages of the NRG method in its low energy resolution

and its dealing with real frequency directly, we obtain for the first time an accurate form for self-energies over the entire energy range for models with both local and nonlocal interactions. We show that the self-energy for Anderson orbitals becomes singular (with a pole) near the Fermi energy in certain regimes controlled by inter-site spin exchange interaction (RKKY interaction). To address the origin of this singularity, we calculate the spectra of composite fermions,  $d_{i\sigma} \equiv f_{i\sigma}n_{i\bar{\sigma}}$  and  $e_{i\sigma} \equiv f_{i\sigma}(1 - n_{i\bar{\sigma}})$ , where  $f_{i\sigma} = d_{i\sigma} + e_{i\sigma}$  is the fermion operator for Anderson orbitals. The two composite fermion operators differentiate the single-particle excitations when the orbital is already single-occupied or vacant. We realize that when these two composite fermions have different behaviors, it is necessary to incorporate their anti-bonding operator  $\bar{f}_{i\sigma} = d_{i\sigma} - e_{i\sigma} = f_{i\sigma}(2n_{i\bar{\sigma}} - 1)$  to account for the single-particle excitations. We find that in the Kondo regime dominated by onsite Kondo coupling, with a Kondo resonance peak in  $f_{i\sigma}$ , the excitations of  $\bar{f}_{i\sigma}$  have a pseudogap. In certain regime dominated by RKKY interaction, where there is a pseudogap in excitations of  $f_{i\sigma}$ , the excitations of  $\bar{f}_{i\sigma}$  has finite spectral weight. We argue  $\bar{f}_{i\sigma}$ , as a canonical fermion, could develop a Fermi surface with Fermi liquid behaviors but “hidden” from single charge excitations. The scattering from this composite fermion leads to a pseudogap (or singularity in self-energy) of  $f_{i\sigma}$ . We further show that a Cooper pair excitation is finite only when both excitations of  $f_{i\sigma}$  and  $\bar{f}_{i\sigma}$  are finite, implying that the mechanism of superconducting pairing may rely on the composite fermion excitations in interacting systems.

**Model and Method.** We consider the Hamiltonian for the two-impurity Anderson model:

$$H = \sum_{\mathbf{k}\sigma} (\epsilon_{\mathbf{k}} - \mu) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \frac{1}{\sqrt{N_L}} \sum_{\mathbf{k}\sigma i} (V_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{r}_i} c_{\mathbf{k}\sigma}^\dagger f_{i\sigma} + h.c.) + \sum_{i\sigma} (\epsilon_f - \mu) f_{i\sigma}^\dagger f_{i\sigma} + \sum_i U n_{fi\uparrow} n_{fi\downarrow}, \quad (1)$$

Here  $c_{\mathbf{k}\sigma}$  and  $f_{i\sigma}$  ( $i = 1, 2$ ) are electron operators conduction band and Anderson orbitals, with respective band dispersion  $\epsilon_{\mathbf{k}}$  and energy level  $\epsilon_f$ . The electrons on the local orbitals

experiences an onsite Coulomb repulsion  $U$ , while the non-interacting electron band hybridizes with local  $f$ -electrons on each site with the strength  $V_{\mathbf{k}}$ .  $\mu$  is the chemical potential. In applications to cluster-DMFT,  $f_{i\sigma}$  are chosen from neighboring sites in bipartite sublattices with  $c_{\mathbf{k}\sigma}$  describing an effective fluctuating bath. In terms of the even or odd parity combinations of the local orbitals,  $f_{p=(e,o)\sigma} = (f_{1\sigma} \pm f_{2\sigma})/\sqrt{2}$ , the bath spectral spectrum  $\Delta_{e,o}(\omega) = \sum_{\mathbf{k}} (V_{\mathbf{k}}^2/2N_L) |e^{i\mathbf{k}\cdot\mathbf{r}_1} \pm e^{-i\mathbf{k}\cdot\mathbf{r}_2}|^2 / (\omega - \epsilon_{\mathbf{k}} + i0^+)$ , is to be determined from a self-consistent procedure. In this case, the even and odd parity states correspond to the momentum points  $\Gamma = (0,0,\dots)$  and  $M=(\pi,\pi,\dots)$  for a  $d$ -dimensional lattice, respectively.

We adopt the NRG method to solve this model. The calculation details can be found in Ref. 8. In particular, we carry out calculations of the self-energies for Anderson orbitals  $\Sigma_{p\sigma}(\omega)$  from the equation of motion,

$$G_{f,p\sigma}^{-1}(\omega) = \omega + \mu - \epsilon_f - \Delta(\omega) - \Sigma_{p\sigma}(\omega),$$

$$\Sigma_{p\sigma}(\omega) = \langle\langle [f_{p\sigma}, H_{f,int}]; f_{p\sigma}^\dagger \rangle\rangle / G_{f,p\sigma}(\omega). \quad (2)$$

As  $H_{f,int} = \sum_i U n_{fi\uparrow} n_{fi\downarrow}$ ,  $[f_{p\sigma}, H_{f,int}]/U = d_{p\sigma} = (f_{1\sigma} n_{1\bar{\sigma}} \pm f_{2\sigma} n_{2\bar{\sigma}})/\sqrt{2}$  is a composite fermion operator.  $F_{p\sigma}(\omega) = \langle\langle [f_{p\sigma}, H_{f,int}]; f_{p\sigma}^\dagger \rangle\rangle$  is usually calculated by a mean-field cutoff in analytical methods, but can be determined exactly in numerics, as firstly introduced for the single-impurity Anderson model [10]. In NRG, the imaginary parts of dynamical quantities are directly calculated with the Lehmann representation. The real parts can be determined subsequently from the Kramers-Kronig relation.

**Results of self-energies.** We start from a presumed form of the bath spectrum,  $\Gamma_{e,o}(\omega) = -\text{Im}\Delta(\omega) = \Gamma_0$  for  $|\omega| \leq D$ , where  $D$  is the bath electron bandwidth and is set as the energy unit. This case has been studied earlier in Refs. 11 and 8. We follow the same numerical procedure and adopt the same parameters  $\Gamma_0 = 0.045\pi$ ,  $U = -2\epsilon_f = 2$  and  $\mu = 0$  as in Ref. 8 [as Case (i)]. As the generated RKKY interaction vanishes for this spectrum, we add an explicit intersite spin exchange term  $IS_1 \cdot S_2$  to simulate the RKKY interaction effect. It is found that for  $I < I_c \approx 2.3T_K^0$ ,  $T_K^0$  as the single-ion Kondo temperature, the ground state is a Kondo resonance state with finite spectrum  $A_f(\omega) = -\text{Im}G_f(\omega)$  at  $\omega = 0$  but with reduced Fermi liquid temperature; for  $I > I_c$ , the ground state is dominated by the RKKY interaction with vanishing spectrum weight  $A_f(\omega) \sim \omega^2$ . At the quantum critical point (QCP)  $I = I_c$ , various correlation functions are found to be divergent, including the staggered magnetic susceptibility, the inter-site singlet Cooper pair correlation function, and the current fluctuation between two sites [11, 12]. In Fig. 1, we show the results of self-energies for different values of  $I$  [See also Fig. 3 for  $A_f(\omega)$ ]. In the Kondo regime ( $I < I_c$ ), it is found that the self-energy has an analytical form  $\text{Re}\Sigma_{p\sigma}(\omega) = U/2 - \omega/Z$  [See inset of Fig. 1(a)] and  $\text{Im}\Sigma_{p\sigma}(\omega) \sim -\omega^2$  at low energies, while  $Z \rightarrow 0$  approaching the QCP. In the RKKY regime ( $I > I_c$ ), where a pseudogap form for the single-particle spectra is identified, we find  $\text{Re}\Sigma_{p\sigma}(\omega) = Z'/\omega$  with  $Z' \rightarrow 0$  approaching the QCP, i.e.,

the self energy has a pole at the Fermi energy. [We experience 0/0 situation in determining  $\text{Im}\Sigma_{p\sigma}(\omega)$  at  $\omega \rightarrow 0$ , whose exact form at low energies relies on numerical precision beyond current calculation. From  $\text{Re}\Sigma_{p\sigma}(\omega) \sim 1/\omega$ , we expect  $\text{Im}\Sigma_{p\sigma}(\omega) \sim \text{const.}$ ]. Apparently, the development of a pseudogap is associated with this singularity in the self-energy, as both  $\text{Re}G_{p\sigma}(\omega)$  and  $\text{Im}G_{p\sigma}(\omega)$  vanish at the energy where  $\text{Re}\Sigma_{p\sigma}(\omega)$  becomes infinite.

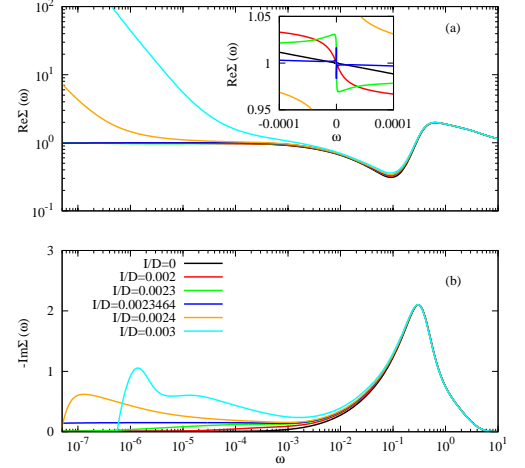


FIG. 1: (Color online) Real (a) and imaginary (b) parts of the self-energy for  $f$ -electrons for the case of  $\Gamma_{e,o}(\omega) = \Gamma_0$  with added  $IS_1 \cdot S_2$  term. Here  $\Gamma_0 = 0.045\pi$ ,  $U = -2\epsilon_f = 2$  and  $\mu = 0$ . Various quantities for different parity and spin channels are the same due to the symmetry. Additional particle-hole symmetry is also preserved. In the inset of panel (a), we show the zoom-in region near the Fermi energy in linear-linear scale to demonstrate the behavior of  $\text{Re}\Sigma$  in the Kondo regime.

In Fig. 2, we further show the self-energies for another case with the bath spectrum determined from a realistic 3D tight-binding dispersion for conduction electrons. Here, it is found that  $\Gamma_{e,o}(\omega)$  have the form of  $\Gamma_0(1 \mp \omega)$  at low energies, and a finite antiferromagnetic RKKY interaction is perturbatively generated [8]. Therefore, we turn off the explicit spin exchange term  $IS_1 \cdot S_2$ , and simply change the value of the hybridization constant  $V_{\mathbf{k}}$  (with  $U$  fixed) to tune the relative strength between  $I$  and  $T_K^0$ . Here the results are for  $U = 2$ ,  $\epsilon_f = -0.5$ , and  $\mu = -0.2$ . Due to the lift of the symmetry between even and odd parities, there is no sharp transition in this case. Instead, the system changes smoothly from a Kondo resonance state to an inter-impurity singlet state, respectively with finite and almost vanishing single-particle spectral weights near the Fermi energy. We also identify the singularity in the self-energy of  $f$ -electrons in certain range of the RKKY dominating regime. In this case, the singularity (as a pole) only exists in the even-parity channel near the Fermi energy.

**Composite fermion excitations.** From Eq. (2), the emergence of a pole in the self-energy takes place when both  $\text{Re}G_{p\sigma}(\omega)$  and  $\text{Im}G_{p\sigma}(\omega)$  vanish while  $F_{p\sigma}(\omega)$  is finite. This motivates us to study the behaviors of separate composite

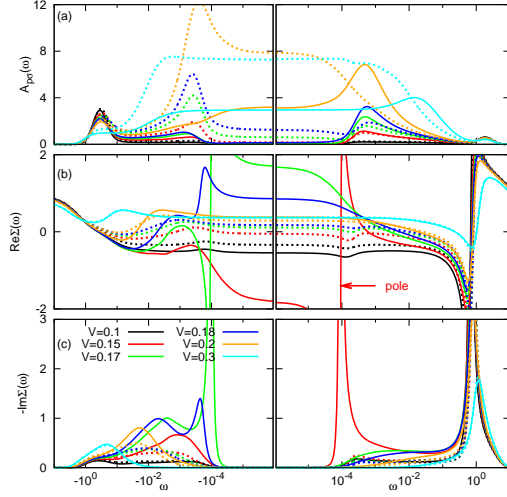


FIG. 2: (Color online) (a) Spectral function  $A_f(\omega)$ , (b) real and (c) imaginary parts of the self-energy for different values of the hybridization constant  $V$  for a realistic band model. Here  $U = 2$ ,  $\epsilon_f = -0.5$ , and  $\mu = -0.2$ . As  $V$  increases, the relative ratio between the generated RKKY interaction  $I$  and  $T_K^0$  decreases, and the system evolves from the RKKY regime to the Kondo regime. The solid and dashed lines represent the even and odd parity channels, respectively. The spin symmetry is preserved. The pole position in  $\text{Re}\Sigma$  for  $V = 0.15$  is illustrated as example, with a  $\delta$ -function-like peak in  $\text{Im}\Sigma$ .

fermion operators  $d_{p\sigma} = [d_{1\sigma} \pm d_{2\sigma}]/\sqrt{2}$  and  $e_{p\sigma} = [e_{1\sigma} \pm e_{2\sigma}]/\sqrt{2}$ . [Notice that  $d_{p\sigma} \neq f_{p\sigma}n_{p\bar{\sigma}}$ , similarly, for  $d_{\mathbf{k}\sigma}$  in a lattice]. In the atomic limit (without the bath),  $f$  electrons only have the upper and lower Hubbard levels, which can be treated as the energy level for  $d_{i\sigma}$  and  $e_{i\sigma}$  composite fermions, i.e., the  $f$ -electron is fractionalized into two flavors of composite fermions. As there is no hybridization between these two composite fermions, the single-particle excitation has a Mott gap, while there is also a pole in self-energy.

In Fig. 3, we show the properties of the composite fermions as well as their hybridizations for the first case. In the Kondo regime ( $I = 0.002$ ), it is found that  $\text{Im}\langle\langle d_{p\sigma}, e_{p\sigma}^\dagger \rangle\rangle \approx \text{Im}\langle\langle d_{p\sigma}, d_{p\sigma}^\dagger \rangle\rangle \approx \text{Im}\langle\langle f_{p\sigma}, f_{p\sigma}^\dagger \rangle\rangle/4$  at  $\omega = 0$ . Together with both vanishing  $\text{Re}\langle\langle d_{p\sigma}, e_{p\sigma}^\dagger \rangle\rangle$  and  $\text{Re}\langle\langle d_{p\sigma}, d_{p\sigma}^\dagger \rangle\rangle$ , this implies that the hybridization between  $d_{p\sigma}$  and  $e_{p\sigma}$  is the coherent part of the single-electron excitation  $f_{p\sigma}$ , which behaves as a non-interaction electron with a renormalized energy level at the Fermi energy. Interestingly, their anti-bonding part  $\bar{f}_{p\sigma} = d_{p\sigma} - e_{p\sigma}$  has a pseudogap form,  $A_{\bar{f}}(\omega) = -\text{Im}G_{\bar{f}}(\omega) \sim \omega^2$ . It can be easily verified that  $\bar{f}_{p\sigma}$  is a canonical fermion, satisfying the anti-commutation rule  $[\bar{f}_{p\sigma}, \bar{f}_{p'\sigma'}^\dagger]_+ = \delta_{pp'}\delta_{\sigma\sigma'}$  [the same for the site basis]. In the RKKY regime ( $I = 0.003$ ), where  $G_f(\omega)$  has a pseudogap,  $A_f \sim \omega^2$ , we find that  $A_{\bar{f}}(\omega)$  is finite near the Fermi energy. The latter resembles a “Kondo resonance” but with a smaller value for  $A_{\bar{f}}(0)$  [It is found to increase as  $V/U$  increases but its exact dependence is to be determined]. In this case, we find that  $\text{Im}\langle\langle d_{p\sigma}, e_{p\sigma}^\dagger \rangle\rangle$  has an

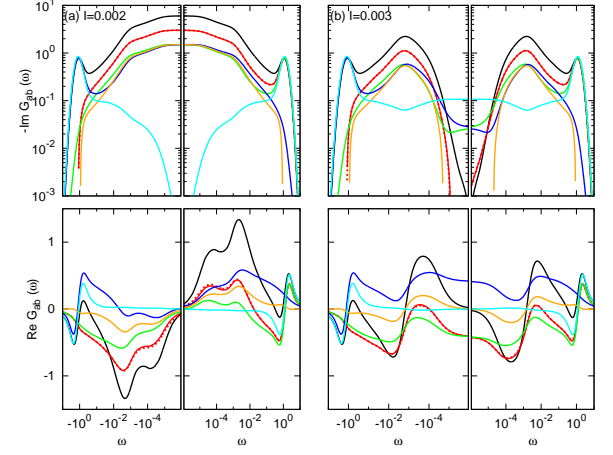


FIG. 3: (Color online) Imaginary and real parts of different composite fermion Green's functions. This is corresponding to the case in Fig. 1 for two values of  $I = 0.002$  (a) and  $I = 0.003$  (b). Lines with different colors represent different Green's functions:  $G_f = \langle\langle f_{p\sigma}, f_{p\sigma}^\dagger \rangle\rangle$  (black),  $\langle\langle d_{p\sigma}, f_{p\sigma}^\dagger \rangle\rangle$  (red) [for comparison  $F_{p\sigma}\omega/U$  (red dotted) with additional contributions from  $IS_1 \cdot S_2$  term],  $\langle\langle d_{p\sigma}, d_{p\sigma}^\dagger \rangle\rangle$  (green),  $\langle\langle e_{p\sigma}, e_{p\sigma}^\dagger \rangle\rangle$  (blue),  $\langle\langle d_{p\sigma}, e_{p\sigma}^\dagger \rangle\rangle$  (orange), and  $\langle\langle \bar{f}_{p\sigma}, \bar{f}_{p\sigma}^\dagger \rangle\rangle$  (cyan).  $-\text{Im}\langle\langle d_{p\sigma}, e_{p\sigma}^\dagger \rangle\rangle$  is negative in the energy range  $|\omega| > U/2$ , and additionally near  $\omega = 0$  for  $I = 0.003$  (not shown due to the log plot). Similarly,  $-\text{Im}\langle\langle d_{p\sigma}, f_{p\sigma}^\dagger \rangle\rangle$  is also negative in certain energy ranges.

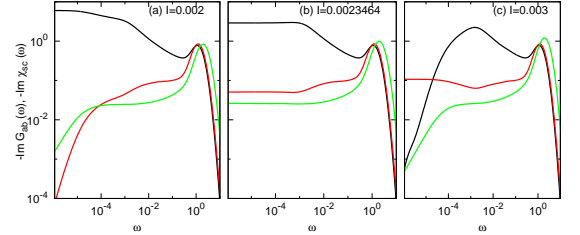


FIG. 4: (Color online) The imaginary part of the Cooper pair correlation function  $\chi_{sc}$  (green lines), the single-particle Green's function  $G_f$  (black lines), and the Green's function  $G_{\bar{f}}$  for the composite fermion  $\bar{f}_{p\sigma}$  (red lines) for three representing values of RKKY interaction in  $I < I_c$ ,  $I \approx I_c$ , and  $I > I_c$ .

opposite sign with  $\text{Im}\langle\langle d_{p\sigma}, d_{p\sigma}^\dagger \rangle\rangle$ , the opposite to the Kondo regime. In addition,  $\text{Re}\langle\langle d_{p\sigma}, d_{p\sigma}^\dagger \rangle\rangle$  and  $\text{Re}\langle\langle e_{p\sigma}, e_{p\sigma}^\dagger \rangle\rangle$  take finite values of the order of  $\mp 1/U$ . This is a reminiscent of the atomic limit result, where, e.g.,  $\langle\langle d_{p\sigma}, d_{p\sigma}^\dagger \rangle\rangle = \langle n_{p\bar{\sigma}} \rangle / (\omega - \epsilon_f - U)$  but with  $\text{Im}\langle\langle d_{p\sigma}, e_{p\sigma}^\dagger \rangle\rangle = 0$ . Here  $\text{Im}\langle\langle d_{p\sigma}, e_{p\sigma}^\dagger \rangle\rangle$  is rather finite at  $\omega \approx 0$ , serving as the difference between a pseudogap and a fully-opened Mott gap. The similar features can also be identified in the realistic band case, as shown in Fig. 2, but we find that away from single-occupancy, we need to redefine  $(g_{p\sigma}, \bar{g}_{p\sigma}) = \alpha d_{p\sigma} \pm \beta e_{p\sigma}$  with  $\alpha/\beta \neq 1$  to capture the above duality (not shown).

We proceed to study the superconducting instability. In Fig. 4, we show the Cooper pair correlation functions  $\chi_{sc} = \langle\langle \Delta_{sc}; \Delta_{sc}^\dagger \rangle\rangle$ , where  $\Delta_{sc}^\dagger = f_{1\uparrow}^\dagger f_{2\downarrow}^\dagger + f_{2\uparrow}^\dagger f_{1\downarrow}^\dagger$ , together with the

single particle spectra for  $f_{p\sigma}^\dagger$  and  $\bar{f}_{p\sigma}^\dagger$ . We notice that the finite pair excitations are closely related to the existence of both finite excitations of  $f_{p\sigma}^\dagger$  and  $\bar{f}_{p\sigma}^\dagger$ . In the particle-hole symmetric case, both  $A_f(0)$  and  $A_{\bar{f}}(0)$  are finite only at the QCP. But in the particle-hole asymmetric (or doped) case, they can be both finite in a range of tuning parameters.

*Discussions.* As illustrated by the composite fermion excitations, the difference between a Kondo resonance state and a pseudogap state of  $f_{p\sigma}$  can be understood as how the excitations of  $d_{p\sigma}$  and  $e_{p\sigma}$  are combined to form quasiparticles. In the Kondo regime, the elementary excitation is predominantly in the form of the regular electron operator  $f_{p\sigma}$ . In the RKKY regime, it is rather in the form of an antibonding operator,  $\bar{f}_{p\sigma} = d_{p\sigma} - e_{p\sigma}$ .  $\bar{f}_{p\sigma}$ , as a canonical fermion, is expected to develop a Fermi surface in the lattice generalization (or participate in forming the Fermi surface in heavy fermion problem). This also accounts for the Fermi liquid behaviors in the RKKY regime, with a finite Fermi liquid temperature vanishing at the QCP [13]. But as the factor  $\langle 2n_{i\bar{\sigma}} - 1 \rangle \approx 0$  in the single-occupancy limit, this composite Fermi surface is not sensitive to charge excitations involving single-particle excitations, such as current, electron tunneling. But it will contribute to spin dynamics and thermodynamics. In this sense, it can be dubbed as a “hidden”-Fermi surface. We notice that it is quite different from similar proposals from other theoretical aspects, such as the spinon Fermi surface. It does not rely on spin-charge separation and it is a Fermi liquid fixed point. We also notice that the composite fermion is in general itinerant, different from local moment pictures.

On the other hand, the composite fermion operator  $\bar{f}_{p\sigma}$  can be treated as the original electron fermion operator combined with a bilinear electron operator capturing different particle-particle or particle-hole excitations, including charge and spin fluctuations. Such fluctuations have been treated as the bosonic fluctuations coupled to electrons. But such approaches, emphasizing on low energy spin fluctuations, could never capture accurately the excitations of  $\bar{f}_{p\sigma}$ . A bosonic fluctuation formalism could not produce a pole in the self-energy, but rather certain singular non-Fermi liquid form. In our calculations, all fluctuations, including Cooper pair fluctuations and current fluctuations are captured exactly and inherently. We notice a recent proposal of composite fermion theory [14], based on the Kotliar-Ruckenstein slave boson approach, which can also capture the charge fluctuations and produce a pole in self-energy in terms of the hybridization between the composite fermion and the quasiparticles. Although we believe that there is a connection between their composite fermions with  $\bar{f}_{p\sigma}$  here, a future study to establish the exact relation is highly demanded. This composite fermion operator can be thought of as a dual representation of the original electron operator, i.e., the original Hamiltonian can be represented by  $\bar{f}_{p\sigma}$  only. The objective to introduce  $d_{p\sigma}$  and  $e_{p\sigma}$  is to eliminate the interaction term with quartic fermion operators. Therefore, as long as the excitations of  $d_{p\sigma}$  and  $e_{p\sigma}$  have different behaviors, it is necessary to incorporate  $\bar{f}_{p\sigma}$  in an effective low-energy Hamiltonian in terms of bilinear electron operators.

We notice that, as  $\bar{f}_{e\sigma} = f_{e\sigma}(n_{e\bar{\sigma}} + n_{o\bar{\sigma}} - 1) + f_{e\bar{\sigma}}^\dagger f_{o\bar{\sigma}} f_{o\sigma} - f_{e\bar{\sigma}} f_{o\bar{\sigma}}^\dagger f_{o\sigma}$ , various charge and spin fluctuations behave as a “hybridization” term  $f_{e\sigma}^\dagger \bar{f}_{e\sigma}$ . However, due to the nontrivial anti-commutation relation between  $f_{i\sigma}$  and  $\bar{f}_{i\sigma}$ , it is not obvious how to construct such a hybridization model.

We also find that the composite fermion  $\bar{f}_\sigma$  does not have finite spectral weight in the local moment regime of the pseudogap Anderson impurity model, implying that the coherent excitation here is an inter-site interaction effect. How exactly the intersite spin exchange interaction leads to the sign change in  $\text{Im}\langle\langle d_{p\sigma}, e_{p\sigma}^\dagger \rangle\rangle$ , which directly differentiate the regular fermion or the composite excitations, is to be studied in the future.

In summary, we have proposed a canonical composite fermion, whose excitations are important to elucidate the singularity in self-energy and the pseudogap state of the interacting electrons.

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